

Statistical and dynamical decoupling of the IGM from Dark Matter

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Abstract

The mean mass densities of cosmic dark matter is larger than that of baryonic matter by a factor of about 5 in the Λ CDM universe. Therefore, the gravity on large scales should be dominant by the distribution of dark matter in the universe. However, a series of observations incontrovertibly show that the velocity and density fields of baryonic matter are decoupling from underlying dark matter field. This paper shows our attempts to unveil the physics behind this puzzle. In linear approximation, the dynamics of the baryon fluid is completely governed by the gravity of the dark matter. Consequently, the mass density field of baryon matter $\rho_b(\mathbf{r}, t)$ will be proportional to that of dark matter $\rho_{\text{dm}}(\mathbf{r}, t)$, even though they are different from each other initially. In weak and moderate nonlinear regime, the dynamics of the baryon fluid can be sketched by Burgers equation. A basic feature of the Burgers dynamics is to yield shocks. When the Reynolds number is large, the Burgers fluid will be in the state of Burgers turbulence, which consists of shocks and complex structures. On the other hand,

the collisionless dark matter may not show such shock, but a multivalued velocity field. Therefore, the weak and moderate nonlinear evolution leads to the IGM-dark matter deviation. Yet, the velocity field of Burgers fluid is still irrotational, as gravity is curl-free. In fully nonlinear regime, the vorticity of velocity field developed, and the cosmic baryonic fluid will no longer be potential, as the dynamics of vorticity is independent of gravity and can be self maintained by the nonlinearity of hydrodynamics. In this case, the cosmic baryon fluid is in the state of fully developed turbulence, which is statistically and dynamically decoupling from dark matter. This scenario provides a mechanism of coherent explanation of observations.

1 Introduction

The mass field of the universe is dominated by dark matter, and only a tiny fraction of cosmic matter is in the form of baryonic particles. Gravitational clustering of dark matter is the major inputs of cosmic structure formation. Collapsed massive halos of dark matter are believed to be the hosts of various observed objects. On the other hand, light-emitting and absorbing objects are made up of baryonic matter.

Most of the baryonic matter in the universe actually is made of intergalactic gas, or intergalactic medium (IGM), mainly consisting of neutral and ionized hydrogen, neutral and ionized helium [1, 2, 3]. For a long time, the mass field of the IGM, $\rho_b(\mathbf{r})$, is believed to be proportional to dark matter field $\rho_{\text{dm}}(\mathbf{r})$. The similarity relation $\rho_b(\mathbf{r}) \propto \rho_{\text{dm}}(\mathbf{r})$ should be hold, at least, on scales larger than the thermal diffusion or Jeans length of the IGM. In other words, the IGM mass field is considered as a point-by-point tracer of the dark matter distribution. Obviously, in this case, the baryon fraction $\rho_b(\mathbf{r})/[\rho_{\text{dm}}(\mathbf{r}) + \rho_b(\mathbf{r})]$ is independent of \mathbf{r} , and equals to the cosmic mean $\Omega_b/(\Omega_b + \Omega_{\text{dm}}) = 0.17 \pm 0.01$ [4, 5], where Ω_b and Ω_{dm} are, respectively, the cosmological parameters of baryonic and dark matter

However, the similarity $\rho_b(\mathbf{r}) \propto \rho_{\text{dm}}(\mathbf{r})$, is found to be seriously inconsistent with observation in collapsed objects. The baryon fractions in gravitational bound objects generally are much lower than the cosmic mean. It raises the so-called missing baryon problem. The baryon fraction is claimed to decrease monotonically with the mass of collapsed halos. For X-ray clusters and groups, the baryon fractions are smaller than the cosmic mean by a factor of 2 - 4 [6, 7, 8]. The baryon fraction of dwarf galaxies can be as low as about 1% of the cosmic mean [9].

Many works have been carried out to explain the deficit of baryon content in gravitational collapsed objects, including feedback due to supernovae [10]

or Active Galactic Nuclei (AGN) [11, 12], which blows out hot gas from halos, and pre-heating [13, 14], which heats baryon gas in the IGM and slows down the accretion of the IGM into halos. However, cosmologically hydrodynamic simulations show that these mechanisms fail to produce the baryon fraction in halos revealed by observation [15, 16, 17]

Actually, the baryon-dark matter decoupling probably is due mainly to the nonlinear nature of the hydrodynamics of the velocity and mass fields of IGM. That is, even without extra heating and cooling, the hydrodynamical evolution can already provide mechanisms of separating the baryonic matter from dark matter. A possible dynamical mechanism driving the baryonic matter deviate from dark matter has been addressed in the pioneer work of Shandarin & Zeldovich [18]. As the dark matter particles are assumed to be collisionless, their velocities would be multi-valued at the intersection of trajectories. However, the baryon matter has a single-valued velocity field as a compressible fluid. Discontinuities, such as shocks or complex structures, will develop inevitably in the baryon fluid at these intersections. Shocks would then cause the under-dense of baryonic matter in the post-shock area. On the other hand, dark matter is not affected by the shocks. Thus, discontinuities might prevent baryon matter to follow dark matter. Consequently, the baryon fraction in the collapsed halos would be less than cosmic mean [19].

Therefore, the nonlinear evolution of IGM would be a major reason of the statistical and dynamical decoupling between the cosmic baryon fluid and dark matter. We report the results of this mechanism. This paper will be arranged as follows. §2 is on the linear theory of evolution of IGM perturbations. §3 presents the decoupling in moderate nonlinear regime. §4 studies the fully nonlinear development of the IGM. It shows the cosmic baryon fluid to be turbulent on small scales. §5 gives observation evidences. The conclusions will be in §6.

2 Linear evolution of the perturbations of IGM mass field

2.1 Basic equations

Let us consider a flat universe having cosmic factor $a(t) \propto t^{2/3}$, and dominated by dark matter. We describe the IGM by a mass density field $\rho(\mathbf{x}, t)$ and a peculiar velocity field $\mathbf{v}(\mathbf{x}, t)$, where \mathbf{x} is the comoving coordinate. In hydrodynamical description, the equations of the IGM consist of the conti-

nunity and the momentum (or the Euler) equations as [20]

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla \cdot (1 + \delta) \mathbf{v} = 0 \quad (1)$$

$$\frac{\partial a \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p - \nabla \phi \quad (2)$$

where the density perturbation of the IGM $\delta(\mathbf{x}, t) = [\rho(\mathbf{x}, t) - \bar{\rho}]/\bar{\rho}$, and $\bar{\rho}$ is the mean density of the IGM. p is the pressure of the IGM.

The IGM can be considered as a "passive substance" with respect to the underlying dark matter. Namely, the evolution of the IGM mass field $\rho(\mathbf{x}, t)$ is governed by the gravity of total matter (baryonic plus dark matter). The peculiar gravitational potential ϕ in eq.(2) satisfies

$$\nabla^2 \phi = 4\pi G a^2 \bar{\rho}_t \delta_t. \quad (3)$$

where the operator ∇ is acting on the comoving coordinate \mathbf{x} , and $\delta_t(\mathbf{x}, t) = [\rho_t(\mathbf{x}, t) - \bar{\rho}_t]/\bar{\rho}_t$ is the perturbation of total density field, with mean density $\bar{\rho}_t = 1/6\pi G t^2 \propto a^{-3}$. The potential ϕ will be zero (or constant) when density perturbation $\delta_t = 0$.

To sketch the gravitational clustering of the IGM, we will not consider the details of heating and cooling. Thermal processes generally are local, and therefore, it is reasonable to describe the thermal processes by a polytropic relation $p(\mathbf{x}, t) \propto \rho^\gamma(\mathbf{x}, t)$. Thus eq.(2) can be rewritten as

$$\frac{\partial a \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\gamma k_B T}{\mu m_p} \frac{\nabla \delta}{(1 + \delta)} - \nabla \phi \quad (4)$$

where the parameter μ is the mean molecular weight of the IGM particles, and m_p is the proton mass. In this case, we don't need energy equation, and the IGM temperature evolves as $T \propto \rho^{\gamma-1}$, or $T = T_0(1 + \delta)^{\gamma-1}$. Exactly speaking, there is also a noise in the hydrodynamical equations eq.(2) and (4), which are related to the viscous of the IGM fluid due to the fluctuation-dissipation theorem. Since the IGM is a "passive substance", this noise would be ignorable in searching for the deviation of the IGM from the underlying dark matter.

2.2 Linear solution of growth mode

The unperturbed solutions of the density and velocity fields of baryonic matter are $\bar{\rho} = [\Omega_b/(\Omega_b + \Omega_{\text{dm}})]\bar{\rho}_t \propto a^{-3}$, and $\mathbf{v} = 0$. Therefore, the linearization of eqs. (1) and (4) yields

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla \cdot \mathbf{v} = 0 \quad (5)$$

$$\frac{\partial a \mathbf{v}}{\partial t} = -\frac{\gamma k_B \bar{T}}{\mu m_p} \nabla \delta - \nabla \phi \quad (6)$$

where the mean temperature $\bar{T} \propto \bar{\rho}^{\gamma-1} \propto a^{-3(\gamma-1)}$. In the Fourier space, we have

$$\frac{\partial^2 \delta(\mathbf{k}, t)}{\partial t^2} + 2\frac{\dot{a}}{a} \frac{\partial \delta(\mathbf{k}, t)}{\partial t} + \frac{1}{t^2} \frac{k^2}{k_J^2} \delta(\mathbf{k}, t) = 4\pi G \bar{\rho} \delta_t(\mathbf{k}, t) \quad (7)$$

$$\frac{\partial v(\mathbf{k}, t)}{\partial t} + \frac{\dot{a}}{a} v(\mathbf{k}, t) = -\frac{1}{t^2 a} \frac{1}{k_J^2} \delta(\mathbf{k}, t) + \frac{4\pi G \bar{\rho} a}{k^2} \delta_t(\mathbf{k}, t), \quad (8)$$

where $\mathbf{v}(\mathbf{k}, t) = i\mathbf{k}v(\mathbf{k}, t)$, and the Jeans wavenumber k_J is given by $k_J^2 = (a^2/t^2)(\mu m_p/\gamma k_B \bar{T}) \propto a^{3\gamma-4}$. When $\gamma = 4/3$, k_J is time-independent. Here, we ignore the stochastic nature of eqs.(7) and (8), and the randomness of all the variables are given by the random initial conditions.

We consider only the growth mode of the perturbation of dark matter, i.e. $\delta(\mathbf{k}, t) \propto a$. In the case of $\gamma = 4/3$, the solution of eqs.(7) and (8) is [21]

$$\delta(\mathbf{k}, t) = \frac{\delta_t(\mathbf{k}, t)}{1 + 3k^2/2k_J^2} + c_1 t^{-(1+\epsilon)/6} + c_2 t^{-(1-\epsilon)/6} \quad (9)$$

where $\epsilon = (1 - 4k^2/9k_J^2)^{1/2}$, and constants c_1 and c_2 depend on the initial condition $\delta(\mathbf{k}, 0)$ and $v(\mathbf{k}, 0)$. Therefore, regardless of the initial condition of the IGM, after a long evolution we always have

$$\delta(\mathbf{k}, t) = \delta_t(\mathbf{k}, t), \quad \text{if } k \ll k_J. \quad (10)$$

This means that the evolution of the IGM will completely follow dark matter, and the initial difference between $\delta(\mathbf{k}, t)$ and $\delta_t(\mathbf{k}, t)$ will be forgotten. Eq.(10) yields

$$P_b(k) = P_{\text{dm}}(k), \quad \text{if } k \ll k_J. \quad (11)$$

The power spectrum of IGM field, $P_b(k)$ has to be the same as that of dark matter $P_{\text{dm}}(k)$.

The linear solutions of the IGM have also been studied by using different assumptions of the IGM thermal processes other than that used in eq.(4) [22, 23]. A common feature of these solutions is

$$\delta(\mathbf{k}, t) = (1 + \text{decaying terms})\delta_t(\mathbf{k}, t) + \text{decaying terms} \quad \text{if } k \ll k_J. \quad (12)$$

Only the decaying terms are affected by the initial conditions, and therefore, the solutions of eqs.(10) and (11) hold in general, regardless of the specific assumptions of the IGM thermal processes.

The physical explanation of the solutions (10) and (12) is straightforward. The IGM is only a tiny component in the cosmic mass field and its evolution

is completely governed by the gravity of dark matter in the linear regime. After the decaying of the initial conditions, the IGM should follow the same trajectory as dark matter on scales larger than the Jeans length.

In linear approximation, we also have the linear relation as follows:

$$\delta(\mathbf{x}, t) = -f\nabla \cdot \mathbf{v}(\mathbf{x}, t), \quad (13)$$

where the parameter f depends on dark matter models [24]

3 Moderate nonlinear regime of the IGM evolution

3.1 Burgers' equation for cosmic baryonic fluid

In the moderate regime, we may use linear relation eq.(13) for δ in eq.(4). We have then

$$\frac{\partial a\mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nu}{a}\nabla^2\mathbf{v} - \nabla\phi \quad (14)$$

where the coefficient ν is given by

$$\nu = \frac{\gamma k_B T_0}{\mu m_p (d \ln D(t)/dt)}, \quad (15)$$

in which $D(t)$ describes the linear growth behavior. The term with ν in eq.(14) describes a diffusion which is characterized by the Jeans length k_J .

To sketch the evolution of gravitational clustering, only the growth modes are of interest. For the growth modes, the velocity field is irrotational, and we can define a velocity potential by

$$\mathbf{v} = -\frac{1}{a}\nabla\phi. \quad (16)$$

Substituting eq.(16) into eq.(14), we have

$$\frac{\partial\phi}{\partial t} - \frac{1}{2a^2}(\nabla\phi)^2 - \frac{\nu}{a^2}\nabla^2\phi = \phi. \quad (17)$$

Equation (17) is the stochastic-force-driven Burgers' equations or the KPZ equations [25]. It is the simplest nonlinear Langevin equation for growth modes [26, 27, 28]. This equation can also be found with a coarse-graining approximation [29].

Thus, in moderate nonlinear regime, the cosmic baryonic fluid is similar to Burgers' fluid. The second term on the l.h.s. results from convection,

and the third on the l.h.s. describes relaxation of the clustering by diffusion. The term on the r.h.s represents the gravitational potential of dark matter, which provides the initially random perturbations. Clustered structures will develop from the initial seeds via the competition of the convection and diffusion.

3.2 IGM as a Burgers' fluid

The Burgers' equation contains two scales: the dissipation length or the Jeans length $1/k_J$ and the correlation length r_c of the random force, ϕ . The intensity of the random force ϕ can be characterized by the density contrast of dark matter δ_{dm} . Many theoretical works on Burgers fluids have been done [30, 31, 32, 33, 34]. They show that turbulence will develop in Burger fluid when the following Reynolds number is large [33, 35]

$$(k_J r_c)^{2/3} \langle \delta_{\text{dm}}^2 \rangle^{1/3} > 1. \quad (18)$$

This condition corresponds to the system in which the dark matter field has entered the non-linear regime. In the linear regime, $\delta_{\text{dm}} \ll 1$, and therefore, perturbations on scale $r_c > 1/k_J$ will not cause Burgers' turbulence in the IGM. In the moderate nonlinear regime, $\delta_{\text{dm}} \geq 1$, and Burgers' turbulence on scales larger the Jeans length can develop. Since the dark matter field becomes nonlinear first on small scales and then on large scales, one can expect that Burgers' turbulence in the IGM develops from small to large scales.

When Burgers turbulence develops in the IGM, its velocity and mass density fields will be intermittent and contains discontinuities, or shocks. The probability distribution function (PDF) of velocity field is long tailed [33]. Since dark matter is not affected by Burgers turbulence, the IGM velocity field will dynamically decouple from the dark matter field on scales larger than the Jeans length once Burgers turbulence is developed. The intermittence of the IGM mass field, as a result of the Burgers equation, has also been emphasized in [23].

Jeans diffusion will also lead to the decoupling between the velocity fields of the IGM \mathbf{v} and dark matter \mathbf{v}_{dm} . However, the decoupling given by the Jeans diffusion is very different from that results from the Burgers turbulence shocks. For the former, the distribution \mathbf{v} and $\Delta \mathbf{v} \equiv \mathbf{v}(\mathbf{x} + \mathbf{r}) - \mathbf{v}(\mathbf{x} - \mathbf{r})$ is symmetric with respect to the transformation $\mathbf{v}_{\text{dm}} \rightarrow -\mathbf{v}_{\text{dm}}$, i.e., the velocity PDF with $\Delta \mathbf{v} \cdot \mathbf{v}_{\text{dm}}/|\mathbf{v}_{\text{dm}}| > 0$ (acceleration in the direction of $\mathbf{v}_{\text{dm}}/|\mathbf{v}_{\text{dm}}|$) is the same as the PDF with $\Delta \mathbf{v} \cdot \mathbf{v}_{\text{dm}}/|\mathbf{v}_{\text{dm}}| < 0$ (deceleration in the direction of $\mathbf{v}_{\text{dm}}/|\mathbf{v}_{\text{dm}}|$). For Burgers turbulence, the shocks consist of an acceleration

ramp followed by a rapid deceleration. Therefore, the IGM is not symmetric between the sections accelerating and decelerating. Since the acceleration is due to the gravity of dark matter, the acceleration is generally in the direction of \mathbf{v}_{dm} . Thus, the IGM velocity field \mathbf{v} is asymmetric with respect to the transformation $\mathbf{v}_{\text{dm}} \rightarrow -\mathbf{v}_{\text{dm}}$. The PDF of $\Delta\mathbf{v}$ with $\Delta\mathbf{v} \cdot \mathbf{v}_{\text{dm}}/|\mathbf{v}_{\text{dm}}| > 0$ will not be the same as the PDF of $\Delta\mathbf{v}$ with $\Delta\mathbf{v} \cdot \mathbf{v}_{\text{dm}}/|\mathbf{v}_{\text{dm}}| < 0$.

If $\nu = 0$, Burgers equation (17) is scale free when the random force ϕ is scale-free. In this case, the velocity field of the Burgers fluid is self-similar and the PDF of the velocity difference $\Delta\mathbf{v}$ is scale-invariant.

In summary, if the IGM can be described as a Burgers' fluid, its velocity field should show the following properties: 1.) the peculiar velocity of the IGM at a given point will generally be lower than that of dark matter at the same point, 2.) the PDF of $\Delta\mathbf{v}_r \cdot \mathbf{v}_{\text{dm}}/|\mathbf{v}_{\text{dm}}| > 0$ will not be the same as the PDF of $\Delta\mathbf{v}_r \cdot \mathbf{v}_{\text{dm}}/|\mathbf{v}_{\text{dm}}| < 0$, and 3.) The PDF of $\Delta\mathbf{v}_r$ is scaling. These predictions have been proven by cosmologically hydrodynamic simulation samples [36].

4 Fully developed nonlinear regime

4.1 Vorticity

Burgers turbulence is qualitatively different from turbulence described by the Navier-Stokes equations. The latter generally consists of vortices on various scales, while the former is a collection of shocks. These features arise because for growth modes of the perturbations of velocity fields is potential, and the velocity field is irrotational. Moreover, from the condition eq.(18), we see that the Burgers turbulence can only develop in dense areas of dark matter $\delta_{\text{dm}} > 1$.

However, in fully developed nonlinear regime, the cosmic baryon fluid will on longer be potential, and the turbulence of Navier-Stokes fluid can also be developed in high as well as low density regions. This point can be seen with the vorticity, which is defined by $\omega_i = (1/2)(\partial_i v_j - \partial_j v_i)$ or $\vec{\omega} = \nabla \times \mathbf{v}$, where $i = 1, 2, 3$. The dynamical equation of the vorticity $\vec{\omega}$ of cosmic baryon fluid can be derived from Navier-Stokes equations as follows [37]

$$\frac{D\vec{\omega}}{Dt} \equiv \partial_t \vec{\omega} + \frac{1}{a} \mathbf{v} \cdot \nabla \vec{\omega} = \frac{1}{a} (\mathbf{S} \cdot \vec{\omega} - d\vec{\omega} + \frac{1}{\rho^2} \nabla \rho \times \nabla p - \dot{a} \vec{\omega}), \quad (19)$$

where $d = \partial_i v_i$ is the divergence of the velocity field. Tensor \mathbf{S} is called strain rate defined as $S_{ij} = (1/2)(\partial_i v_j + \partial_j v_i)$, and the vector $[\mathbf{S} \cdot \vec{\omega}]_i = S_{ij} \omega_j$.

A remarkable property of eq.(19) is the absence of the term of gravity of mass fields, indicating that the gravitational field of dark matter is not

the sources of the vorticity. This point is obvious, as gravity is curl-free in nature. In this sense, the vorticity is "non-gravitational".

In the linear regime of perturbations, both the terms $\mathbf{S} \cdot \vec{\omega} - d\vec{\omega}$ and $\nabla\rho \times \nabla p$ would be zero. Vorticity $\vec{\omega}$ will decay as a^{-1} , given by the term $\dot{a}\vec{\omega}$. Therefore, as expected, in the linear regime the velocity field of baryon matter is irrotational. In other words, the linear evolution of the velocity field of baryon fluid is fully governed by the gravity.

However, in the fully developed nonlinear regime, the vorticity will no longer be zero. First, it has been shown that at the moderate nonlinear regime, Burgers' fluid will inevitably lead to the development of shocks and complex structures, which can yield non-zero term $\nabla\rho \times \nabla p$ of eq.(19), being called baroclinity. Thus, the vorticity is excited, regardless of the gravity of dark matter. Physically, once multi-streaming has developed, complex structures, like curved shocks, will lead to deviation of the direction of ∇p from that of $\nabla\rho$. Consequently, the density pressure relation cannot be simply given by an single-variable function equation as $p = p(\rho)$ [38] and the baroclinity term will no longer be zero.

Once vorticity is initiated, eq.(19) shows that the vorticity of baryon velocity field can be maintained by itself, i.e. the system actually is nonlinearly self-excited and self-maintained. The divergence $d = \partial_j v_j$ is generally negative in regions of clustering. Therefore, the term $-d\vec{\omega}$ of eq.(19) will lead to an amplification of vorticity in overdense regions. On the other hand, the vorticity will be stretched by the nonlinear term $\mathbf{S} \cdot \vec{\omega}$. Therefore, vorticity can also develop in low mass density region. This picture is very different from the Burgers' turbulence.

Vortices are considered as a fundamental ingredient of turbulence. The fluctuations of the vorticity field is an important measurement of the turbulence of fluid [39, 40]. Therefore, the dynamics of vorticity provides a non-gravitational mechanism of the decoupling between baryon matter from dark matter.

The turbulence of cosmic baryon fluid can also be seen with the spectrum tensors $\Phi_{ij}(\mathbf{k})$ and $\Omega_{ij}(\mathbf{k})$, which are, respectively, the Fourier counterparts of the two-point correlation tensors of velocity $\langle v_i(\mathbf{x} + \mathbf{r})v_j(\mathbf{x}) \rangle$ and vorticity $\langle \omega_i(\mathbf{x} + \mathbf{r})\omega_j(\mathbf{x}) \rangle$. That is

$$\Phi_{ij}(\mathbf{k}) = \frac{1}{(2\pi)^3} \int \langle v_i(\mathbf{x} + \mathbf{r})v_j(\mathbf{x}) \rangle e^{-i\mathbf{k} \cdot \mathbf{r}} d\mathbf{r} \quad (20)$$

$$\Omega_{ij}(\mathbf{k}) = \frac{1}{(2\pi)^3} \int \langle \omega_i(\mathbf{x} + \mathbf{r})\omega_j(\mathbf{x}) \rangle e^{-i\mathbf{k} \cdot \mathbf{r}} d\mathbf{r}. \quad (21)$$

respectively, where $\langle \dots \rangle$ denotes average over spatial coordinates \mathbf{x} . For a homogeneous turbulence, we have [39]

$$\Omega_{ij}(\mathbf{k}) = [\delta_{ij}k^2 - k_i k_j] \Phi_{ll}(\mathbf{k}) - k^2 \Phi_{ij}(\mathbf{k}), \quad (22)$$

and hence,

$$\Omega_{ii}(\mathbf{k}) = k^2 \Phi_{ii}(\mathbf{k}). \quad (23)$$

The power spectra of velocity and vorticity fields are defined respectively as

$$P_v(k) = \int \frac{1}{2} \Phi_{ii}(\mathbf{k}) \delta(|\mathbf{k}| - k) d\mathbf{k}; \quad P_\omega(k) = \int \frac{1}{2} \Omega_{ii}(\mathbf{k}) \delta(|\mathbf{k}| - k) d\mathbf{k}. \quad (24)$$

Combining eqs. (23) and (24) yields

$$P_\omega(k) = k^2 P_v(k). \quad (25)$$

This relation can be used to measure the developed level of turbulence. If the velocity and vorticity fields of a fluid satisfy the relation given by eq.(25), it should be in the state of fully developed homogeneous turbulence. Otherwise, it would be less developed. The samples of cosmological hydrodynamic simulation have showed that the relation eq.(25) is perfectly hold on scales less than $3h^{-1}$ Mpc at $z \sim 0$ [37]. Actually, the dynamics of eq.(19) reveals that the cosmic baryonic fluid underwent a scale-free evolution, which leads to a state of fully developed turbulence.

4.2 She-Leveque intermittence

The velocity field of a fully developed turbulence is highly intermittent, which is measured by the structure functions $S_p(r)$ defined as

$$S_p(r) \equiv \langle \delta v_r^p \rangle \sim r^{\zeta_p}. \quad (26)$$

where $\delta v_r \equiv [(\mathbf{v}(\mathbf{x} + \mathbf{r}) - \mathbf{v}(\mathbf{x})) \cdot \mathbf{r}/r]$ and ζ_p is called intermittent exponent. Based on dimensional argument of hierarchical evolution, Kolmogorov in 1941 [41] predicted that for fully developed turbulence, the intermittent exponent is $\zeta_p = p/3$ on scales of inertial range. Experimental and numerical results do not, however, support the $p/3$ law. It should be attributed to intermittency, i.e. turbulence field in inertial range is characterized by stronger non-Gaussianity on smaller scales. A remarkable development was made by She and Leveque (SL hereafter) [42]. They proposed that the non-Gaussian behavior of fully developed turbulence is determined by the hierarchical structure originated from the Navier-Stokes equation, and the $p/3$ law should be replaced by

$$\zeta_p/\zeta_3 = [1 - C(1 - \beta^3)]p/3 + C(1 - \beta^p), \quad (27)$$

which contains only two dimensionless parameter C and β . C is the Hausdorff dimension of the most dissipative structures, and parameter β is given by the hierarchical evolution. SL formula eq.(27) is in excellent agreement with various experiments of turbulence, including turbulence in compressible fluid. The SL scaling law of structure function is considered to be universal for characterizing the fully developed turbulence.

With the hydrodynamic simulation sample of the concordance Λ CDM universe, the intermittent exponent of the velocity field of cosmic baryon fluid at redshift $z = 0$ in the scale range from the Jeans length to about $16 \text{ h}^{-1} \text{ Mpc}$ is found to be extremely well described by the She-Leveque's universal scaling formula eq.(27) with $C = 1$ and $\beta^3 = 1/3$ [43]. It indicates that the dissipative structures are dominated by sheets. These results imply once again that the evolution of highly evolved cosmic baryon fluid is similar to a fully developed turbulence.

When a fluid is turbulent, the kinetic energy passes from large to the smallest eddies, and finally dissipates into thermal motion. For cosmic baryonic gas, the evolution of vorticity is also hierarchical. The vorticity evolves from large scales to small scales, and finally falls into massive halos to form structures, including light-emitting objects. This result strongly indicates that the highly nonlinear evolution would lead to the cosmic baryon fluid reaching a statistically quasi-equilibrium state satisfying the universal scaling as that of fully developed turbulence.

In view of this picture, we can say that in the highly nonlinear regime, the statistical properties, especially the intermittent behavior, of the velocity fluctuations are actually independent of the details of the dissipative processes. The state depends only on the dimension of dissipative structures and the hierarchical relation index.

5 Observable effects

The nonlinearly evolved IGM provides a coherent and uniform explanation of various observations, including the intermittence and non-Gaussianity of $\text{Ly}\alpha$ transmitted flux of quasar absorption spectrum [44, 45]; $\text{Ly}\alpha$ leaks of absorption spectrum of high redshift objects[46]; the turbulence broadening[47]; Scaling relations between SZ effect and X-ray luminosity of X-ray clusters [38, 48, 49], baryon missing in collapsed halos [19]; etc. Here we address only two of them.

5.1 Non-Gaussianity of Log-Poisson hierarchy

It has been shown that the SL scaling is yielded from the Log-Poisson hierarchy process, which is related to the so-called generalized scale covariance of the Navier-Stokes equations [50]. Therefore, the statistical behavior of the mass field of cosmic baryon matter should be given by the log-Poisson random multiplicative processes (RMP).

The log-Poisson RMP assumes that, in the scale-free range, the variables $\delta\rho_r \equiv \rho(\mathbf{x} + \mathbf{r}) - \rho(\mathbf{x})$, $r = |\mathbf{r}|$, on different scales r are related from each other by a statistically hierarchy relation as

$$|\delta\rho_{r_2}| = W_{r_1 r_2} |\delta\rho_{r_1}|, \quad (28)$$

where

$$W_{r_1 r_2} = \beta^m (r_1/r_2)^\gamma, \quad (29)$$

which describes how the fluctuation $|\delta\rho_{r_1}|$ on the larger scale r_1 related to fluctuations $|\delta\rho_{r_2}|$ on the smaller scale r_2 . In eq.(29), m is a Poisson random variable with the PDF

$$P(m) = \exp(-\lambda_{r_1 r_2}) \lambda_{r_1 r_2}^m / m!. \quad (30)$$

To insure the normalization $\langle W_{r_1 r_2} \rangle = 1$, where $\langle \dots \rangle$ is over m , the mean $\lambda_{r_1 r_2}$ of the Poisson distribution should be

$$\lambda_{r_1 r_2} = \gamma [\ln(r_1/r_2)] / (1 - \beta). \quad (31)$$

It is enough to consider only $|\delta\rho_r|$, as the distribution of positive and negative $\delta\rho_r$ is symmetric.

The log-Poisson hierarchy given by eq.(29) depends only on the ratio r_1/r_2 , which is obviously scale invariant. The hierarchy is determined by two dimensionless positive parameters: β and γ , describing, respectively, the intermittence and singularity of the random fields. Equation (29) relates $\delta\rho_r$ on different scales by multiplying a random factor W , which generally yields a non-Gaussian field even if the field originally is Gaussian [51].

The cascade from scale r_1 to r_2 , and then to r_3 is identical to the cascade from r_1 to r_3 . It is because $W_{r_1 r_3} = W_{r_1 r_2} W_{r_2 r_3} = \beta^N (r_1/r_3)^\gamma$, where N is again a Poisson random variable with $\lambda_{r_1 r_3} = \lambda_{r_1 r_2} + \lambda_{r_2 r_3} = \gamma [\ln(r_1/r_3)] / (1 - \beta)$. Therefore, the log-Poisson hierarchy removes an arbitrariness in defining the steps of cascade from r_1 to r_2 or r_2 to r_3 . The log-Poisson hierarchy is discrete in terms of the discrete random number m . However, the scale r is infinitely divisible. Namely, there is no lower limit on the difference $r_1 - r_2$. It can be infinitesimal, and the hierarchical process is of infinite divisibility.

With the log-Poisson hierarchy eqs.(28)-(31), the density field should have the non-Gaussianities, which can be described by the structure function of density field $S_p(r) \equiv \langle |\delta\rho_r|^p \rangle$ as follows [52]:

1. Intermittent exponent. The structure function should be a function of power law of r as

$$S_p(r) \propto r^{\xi(p)}, \quad (32)$$

and the intermittent exponent $\xi(p)$ is given by

$$\xi(p) = -\gamma[p - (1 - \beta^p)/(1 - \beta)]. \quad (33)$$

This is actually the SL scaling formula for density field.

2. Hierarchical relation. Defining

$$F_p(r) \equiv S_{p+1}(r)/S_p(r), \quad (34)$$

the log-Poisson RMP gives hierarchical relation as

$$\frac{F_p(r)}{F_\infty(r)} = \left[\frac{F_{p+1}(r)}{F_\infty(r)} \right]^{1/\beta}. \quad (35)$$

Relation (35) is invariant with respect to a translation in p .

3. Moments. The ratio between high order and 2nd order moments, $\langle \delta\rho_r^{2p} \rangle / \langle \delta\rho_r^2 \rangle^p$ should satisfy

$$\ln \frac{\langle \delta\rho_r^{2p} \rangle}{\langle \delta\rho_r^2 \rangle^p} = K_p \ln r + \text{const} \quad (36)$$

with

$$K_p = -\gamma \frac{p(1 - \beta^2) - (1 - \beta^{2p})}{1 - \beta}. \quad (37)$$

4. Scale-scale correlation. The so-called scale-scale correlation is defined as

$$C_{r_1, r_2}^{p, p} \equiv \frac{\langle \delta\rho_{r_1}^p \delta\rho_{r_2}^p \rangle}{\langle \delta\rho_{r_1}^p \rangle \langle \delta\rho_{r_2}^p \rangle}. \quad (38)$$

Obviously, for a Gaussian field, $C_{r_1, r_2}^{p, p} = 1$. If the ratio r_2/r_1 is fixed, the log-Poisson model predicts the scale-scale correlation to be

$$C_{r_1, r_2}^{p, p} = B(r_2/r_1) r_1^{\xi(2p) - 2\xi(p)}, \quad (39)$$

where the coefficient $B(r_2/r_1)$ depends only on the ratio r_2/r_1 , as the log-Poisson model is invariant of the dilation.

It should be emphasized again that all above-mentioned non-Gaussianities depend only on two dimensionless parameters β and γ . It has been revealed that the fields of Ly α transmitted flux fluctuations of observed high resolution and high S/N Ly α absorption spectra of quasars at moderate redshift $z \sim 2 - 3$, and high redshift $z \sim 5 - 6$ do show all the log-Poisson non-Gaussianities [53, 54].

5.2 Turbulence pressure and baryon missing

As shown in §2.2, the Jeans length $\lambda_J = 2\pi/k_J$ is $\lambda_J = c_s \sqrt{\pi/G\rho}$, c_s being the sound speed. The effect of turbulent motions on the Jeans length can be estimated by replacing c_s by an effective sound speed

$$c_{s,\text{eff}}^2 = c_s^2 + \frac{1}{3}\langle v^2 \rangle \quad (40)$$

where $\langle v^2 \rangle$ is the rms velocity dispersion due to turbulent motion [55, 56]. Namely, the random velocity field of turbulence contributes to an extra pressure, the turbulence pressure, $p_{\text{turb}} = \rho \langle v^2 \rangle$ to resist the gravitational collapsing.

If the collapsed halo is on scale R , the gravitational collapsing will not be affected by the rms velocity dispersion on scales larger than R , the fluctuation of velocity on the scales $k < 2\pi/R$ do not contribute to the turbulent pressure in terms of resist gravitational collapsing on scales that larger than R . Thus, the effect of turbulent pressure on gravitational clustering should be given by [57]

$$p_{\text{turb}}(k_R) = \int_{k_R}^{k_{\text{max}}} E(k) dk, \quad (41)$$

where $E(k)$ is the power spectrum of kinetic energy of the turbulence, and $k_R = 2\pi/R$. Wavenumber $k_{\text{max}} = 2\pi/l_{\text{diss}}$ corresponds to the minimal scale l_{diss} below which the turbulence decays due to energy dissipation or virialization. The turbulent pressure is dynamical, not thermal. It can be as strong as a thermal pressure of temperature 10^{5-6} K, while the temperature of the IGM is still remained in the range 10^{4-5} K.

Obviously, the bulk velocity will not contribute to the $\langle v^2 \rangle$, as it can be given by the motion of falling into a gravitational well. Therefore, we should pick up the component of velocity field that is regardless of gravity. It is mainly given by the vorticity of velocity field. In other words, the turbulent vorticity field can prevent the collapsing of baryon matter into the halos of dark matter. It leads to the deviation of the mass density fields of baryon matter from that of dark matter.

With the samples produced by cosmological hydrodynamical simulation, we find that the turbulence pressure will play an important role of preventing the IGM clustering. In the IGM regions with 10 times mean overdensity, the turbulent pressure is equivalent to the thermal pressure of the baryon gas with temperature of about 1×10^5 K. The turbulent pressure is dynamical and non-thermal. When the turbulence dissipated, the kinetic energy of turbulence becomes the thermal energy. It yields the entropy in halos [38].

The baryon fraction of dark matter halos is found to be a strong function of the halo mass M_{200} , the mass within the range of total mass density larger than $200\bar{\rho}_t$. The baryon fraction is significantly decreasing with the mass of dark halos. For halos with mass $M_{200} = 3 \times 10^{10} M_\odot$, the baryon fraction is only about 0.1 of the cosmic mean of the concordance Λ CDM universe, and it further decreases to 0.03 for $M_{200} = 10^{10} M_\odot$ halos. This result is well consistent with observed baryon content of galaxies and galaxy groups. Therefore, the turbulent velocity field of baryon fluid would be the major factor leading to the absence of baryon matter in halos with mass less than $10^{11.5} M_\odot$. With this evolution picture, no pre-heating or reheating mechanisms are needed for explaining baryon missing.

6 Conclusion

The nonlinear evolution of cosmic baryon fluid is a central problem of cosmology. The concordance Λ CDM model provides an effective paradigm of formation and evolution of cosmic structures. The dynamical behavior of the CDM component has been successfully revealed with analytical study and N-body numerical simulation. On the other hand, the dynamics of baryon fluid is less understood, especially in the nonlinear regime, because the involvement of complicated physical processes, such as cooling, heating, ionization, shock, turbulence etc. On the other hand, many observations are directly related to the nonlinear evolution of baryon fluid, such as the absorption spectra of quasars, galaxies and GRB at high redshift, Sunyaev-Zeldovich effect, the polarization of cosmic microwave background (CMB), X-ray background etc. One of the biggest challenges in galaxy formation is also on the physical origin of the non-linear relation between baryon gas and low-mass dark halos.

Thanks for the development of reliable cosmological hydrodynamical simulation in the last decade, significant progress has been made in understanding the non-linear dynamics of cosmic baryon fluid. The turbulence behavior of cosmic baryon fluid in scale free regime is the key of understanding nonlinear evolution of the mass density and velocity fields. An important consequence of the turbulence is to yield statistical decoupling of the mass and velocity fields of baryon fluid from the underlying dark matter field. This result is extremely valuable to understand the low baryon fraction in galaxy clusters, the non-Gaussian features of quasar Ly α transmitted flux, the Ly α leaking of absorption spectra of high redshift objects, the shock heating of pre-virialized structures, and the strongly nonlinear dependence of the star formation efficiency on halo mass etc. Considering these complexities, the

nonlinear dynamics of the IGM also is essential for understanding the epoch of reionization and related observations.

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